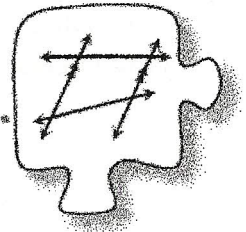


Notes

# 2.1.1 What is the relationship?

## Complementary, Supplementary, and Vertical Angles



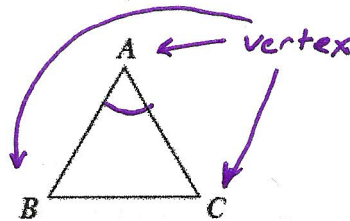
In Chapter 1, you compared shapes by looking at similarities between their parts. For example, two shapes might have sides of the same length or equal angles. In this chapter you will examine relationships between parts within a *single* figure or diagram. Today you will start by looking at angles to identify relationships in a diagram that make angle measures equal. As you examine angle relationships today, keep the following questions in mind to guide your discussion:

How can I name the angle?

What is the relationship?

How do I know

2-2. In this chapter, we will be learning a lot about angles and relationships between them. But in order to clearly describe relationships between angles, you will need a convenient way to refer to and name them. Examine the diagram of equilateral  $\triangle ABC$  below.

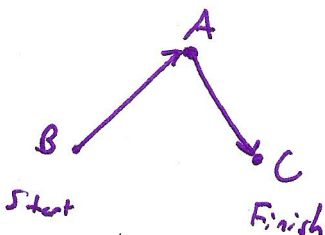


$\angle A \rightarrow$  identify  
 $m\angle A = 60^\circ \rightarrow$  measure

- a. The "top" of this triangle is usually referred to as "angle A," written  $\angle A$ . Point A is called the **vertex** of this angle. The measure of  $\angle A$  (the number of degrees in angle A) is written  $m\angle A$ . Since  $\triangle ABC$  is equilateral, write an equation showing the relationship between its angles. This angle can also be named "angle BAC" written  $\angle BAC$ . Whenever you use 3 letters to name an angle, be sure the vertex is in the middle. When would you need to use 3 letters to name an angle instead of just 1?

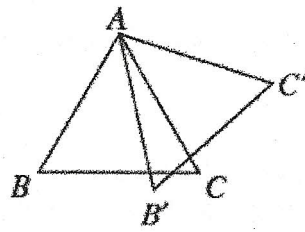
$$m\angle A \cong m\angle B \cong m\angle C$$

$$\angle A = \underline{\angle BAC}$$



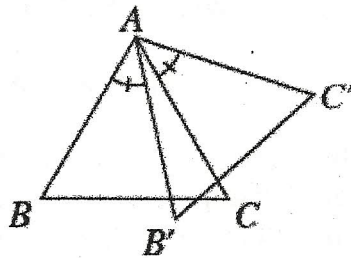
When in doubt use 3 letters to name an angle

- b. Audrey rotated  $\triangle ABC$  around point  $A$  to form  $\triangle AB'C'$ . She told her teammate Maria, "I think the two angles at  $A$  are equal." Maria did not know which angles she was referring to. How many angles can you find at  $A$ ? Are there more than three?



$\angle BAC$   
 $\angle BAB'$      $\angle CAC'$   
 $\angle BAC'$   
 $\angle B'AC$   
 $\angle B'AC'$

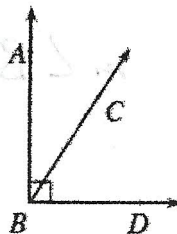
- c. Maria asked Audrey to be more specific. She explained, "One of my angles is  $\angle BAB'$ ." At the same time, she marked her two angles with the same marking below to indicate that they have the same measure. Name her other angle. Be sure to use three letters so there is no confusion about which angle you mean.



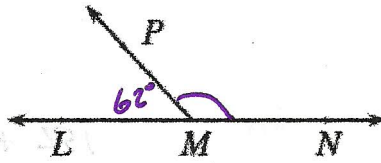
### 2-3. ANGLE RELATIONSHIPS

When you know two angles have a certain relationship, learning something about one of them tells you something about the other. Certain angle relationships come up often enough in geometry that they are given special names.

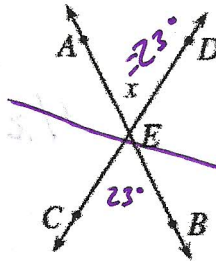
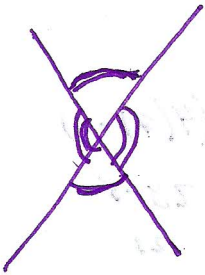
- a. Two angles whose measures have a sum of  $90^\circ$  are called **complementary angles**. Since  $\angle ABD$  is a right angle in the diagram below, angles  $\angle ABC$  and  $\angle CBD$  are complementary. If  $m\angle CBD = 76^\circ$ , what is  $m\angle ABC$ ? Show how you got your answer.



- b. Another special angle is  $180^\circ$ . If the sum of the measures of two angles is  $180^\circ$ , they are called **supplementary angles**. In the diagram below,  $\angle LMN$  is a straight angle. If  $m\angle LMP = 62^\circ$ , what is  $m\angle PMN$ ?



- c. Now consider the diagram below, which shows  $\overline{AB}$  and  $\overline{CD}$  intersecting at  $E$ . If  $x = 23^\circ$ , find  $m\angle AEC$ ,  $m\angle DEB$ , and  $m\angle CEB$ . Show all work.



$m\angle CEB = 23^\circ$   
 $m\angle DEB = 157^\circ$   
 $m\angle AEC = 157^\circ$

- d. Based on your work in part (c), which angle has the same measure as  $\angle AED$ ?

$\angle CEB$

- e. When two lines intersect, the angles that lie on opposite sides of the intersection point are called **vertical angles**. For example, in the diagram above,  $\angle AED$  and  $\angle CEB$  are vertical angles. Find another pair of vertical angles in the diagram.

$\angle AEC$      $\angle DEB$

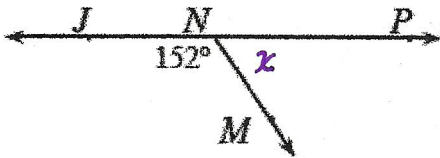
2-4. Travis noticed that the vertical angles in parts (c) and (d) of problem 2-3 have equal measure and wondered if other pairs of vertical angles also have equal measure.

- a. Return to the diagram above and find  $m\angle CEB$  if  $x = 54^\circ$ . Show all work.
- b. Based on your observations, write a **conjecture** (a statement based on an educated guess that is unproven). Start with, "Vertical angles ..."

Vertical angles \_\_\_\_\_

2-5. In the problems below, you will use geometric relationships to find angle measures. Start by finding a special relationship between some of the sides or angles, and use that relationship to write an equation. Solve the equation for the variable, then use that variable value to answer the original question.

a. Find  $m\angle MNP$ .

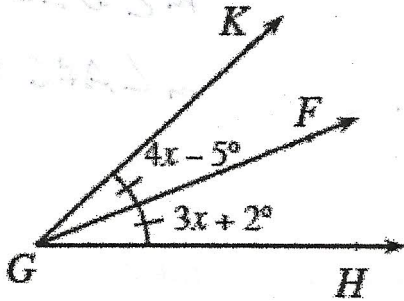


$$152 + x = 180$$

$$x = 28^\circ$$

$$m\angle MNP = 28^\circ$$

b. Find  $m\angle FGH$ .



$$4x - 5 = 3x + 2$$

$$(4(7) - 5)$$

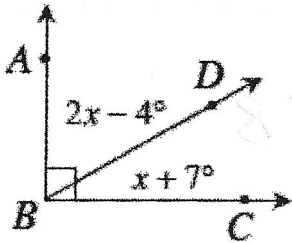
$$x = 7$$

$$28 - 5$$

$$23$$

$$m\angle KGF = m\angle FGH = 23^\circ$$

c. Find  $m\angle DBC$ .



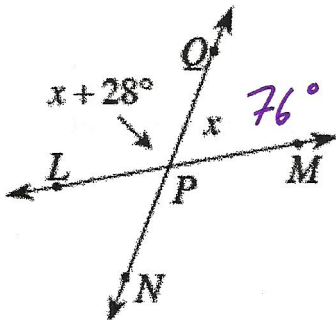
$$2x - 4 + x + 7 = 90$$

$$3x + 3 = 90$$

$$3x = 87$$

$$x = 29$$

d. Find  $m\angle LPQ$  and  $m\angle LPN$ .



$$x + 28 + x = 180$$

$$2x + 28 = 180$$

$$2x = 152$$

$$x = 76$$