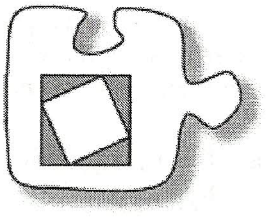
Notes

2.3.2 Is there a shortcut?



The Pythagorean Theorem

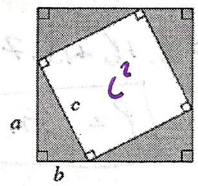
In Lesson 2.3.1, you learned a method to find the length of a hypotenuse of a right triangle by finding the area of the square built on the hypotenuse, as shown in the diagram at right. However, what if the sides of the triangle make it difficult to draw (such as very large numbers or decimal values)? Or what if you do not even know the lengths of one of the legs?

Today, you will work with your team to find the relationship between the legs and hypotenuse of a right triangle. By the end of this lesson, you should be able to find the side of *any*right triangle, when given the lengths of the other two sides.

2-109. Roiri complained that while his method from problem 2-99 works, it seems like too much work! He remembers that rearranging a shape does not change its area and thinks he can find a shortcut. Obtain a Lesson 2.3.2 Resource Page for your team and cut out the shaded triangles. Note that the lengths of the sides of the triangles are *a*, *b*, and *c* units respectively. Explore using the interactive eTool: Pythagorean Theorem (Desmos).

a. First, arrange the triangles to look like Roiri's in the diagram below. Draw this diagram on your paper. What is the area of the unshaded square?

Remember, Area of Square $= \text{side}^2$

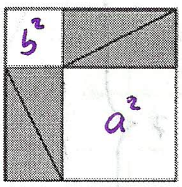


Area $= c^2$

b. Roiri claims that moving the triangles within the outer square won't change the area of the unshaded square. Is Roiri correct? Why or why not?

No, b/c its a translation

c. Move the shaded triangles to match the diagram below. In this configuration, what is the total area that is unshaded?



$a^2 + b^2$

d. Write an equation that relates the two ways that you found to represent the unshaded area in the figure.

$a^2 + b^2 = c^2$

$a^2+b^2=c^2$

2-110. The relationship between the square of the lengths of the legs and the square of the length of the hypotenuse in a right triangle that you found in problem 2-109 is known as the **Pythagorean Theorem**. This relationship is a powerful tool because once you know the lengths of any two sides of a right triangle, you can find the length of the third side.

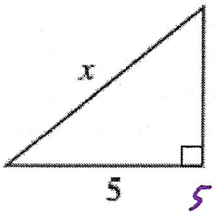
a. Use the dynamic geometry tool, Pythagorean Theorem (Desmos), to examine how the square of the hypotenuse always equals the sum of the squares of the legs of a right triangle. Think about it until it makes sense and you can explain it to someone else so that it will make sense to him or her.

b. LEARNING LOG
Add an entry in your Learning Log for the Pythagorean Theorem, explaining what it is and how to use it. Be sure to include a diagram. Title this entry, "Pythagorean Theorem" and include today's date.

2-111. Apply the Pythagorean Theorem to answer the questions below.

a. For each triangle below, find the value of the variable.

i.



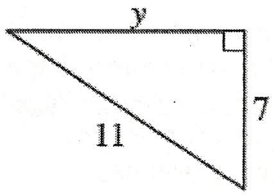
$5^2+4^2=x^2$

$25+16=x^2$

$\sqrt{41}=\sqrt{x^2}$

$\boxed{x=6.4}$

ii.



$y^2+7^2=11^2$

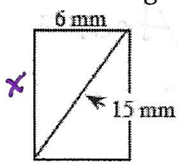
$y^2+49=121$

$-49 \quad -49$

$\sqrt{y^2}=\sqrt{72}$

$\boxed{y=\sqrt{72}\approx 8.5}$

b. Examine the rectangle shown below. Find its perimeter and area.



$6^2+x^2=15^2$

$36+x^2=225$

$-36 \quad -36$

$\sqrt{x^2}=\sqrt{189}$

$x=13.7$

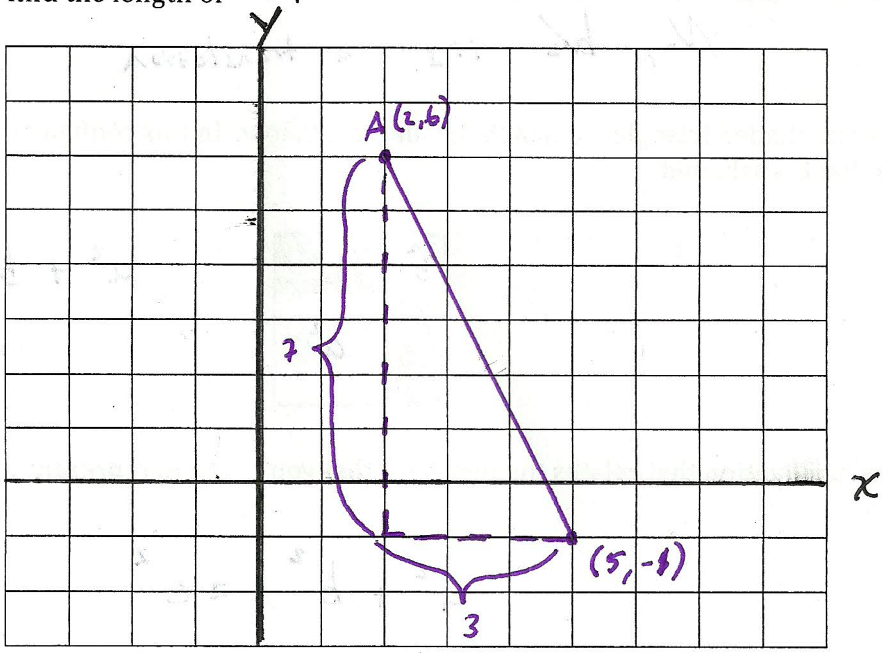
$P=6+6+13.7+13.7$

$\boxed{P=39.4\text{mm}}$

$A=6(13.7)$

$\boxed{A=82.2\text{ mm}^2}$

c. On graph paper, draw $\overline{AC}$ with coordinates $A(2, 6)$ and $C(5, -1)$. Then draw a slope triangle. Use the slope triangle to find the length of $\overline{AC}$.



$7^2+3^2=x^2$

$49+9=x^2$

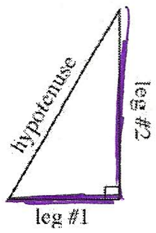
$58=x^2$

$\boxed{x=\sqrt{58}\approx 7.6}$

METHODS AND MEANINGS

MATH NOTES

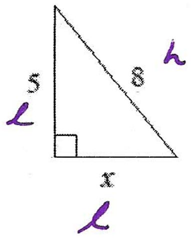
The Pythagorean Theorem



The **Pythagorean Theorem** states that in a right triangle,

(lengthof leg #1)^2 + (length of leg #2)^2 = (length of hypotenuse)^2

The Pythagorean Theorem can be used to find a missing side length in a right triangle. See the example below.



$a^2 + b^2 = c^2$

$$5^2 + x^2 = 8^2$$

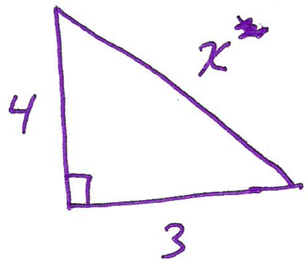
$$25 + x^2 = 64$$

$$-25 \qquad -25$$

$$\sqrt{x^2} = \sqrt{39}$$

$$x = \sqrt{39} \approx 6.24$$

In the example above, $\sqrt{39}$ is an example of an **exact** answer, while 6.24 is an **approximate** answer.



$$3^2 + 4^2 = x^2$$

$$9 + 16 = x^2$$

$$\sqrt{25} = \sqrt{x^2}$$

$$\boxed{5 = x}$$