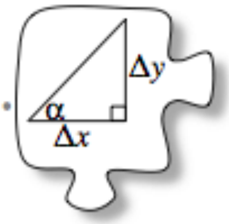
Name: ______________________________ Date: ____________

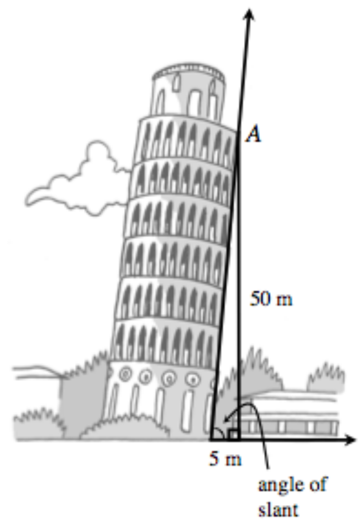
4.1.1 What patterns can I use?

Constant Ratios in Right Triangles



In Chapter 3, you looked for relationships between triangles and ways to determine if they are similar or congruent. Now you are going to focus your attention on slope triangles, which were used in algebra to describe linear change. Are there geometric patterns within slope triangles themselves that you can use to answer other questions? In this lesson, you will look closely at slope triangles on different lines to explore their patterns.

4-1. LEANING TOWER OF PISA



For centuries, people have marveled at the Leaning Tower of Pisa due to its slant and beauty. Ever since construction of the tower started in the 1100s, the tower has slowly tilted south and has increasingly been at risk of falling over. It is feared that if the angle of slant ever falls below 83°, the tower will collapse.

Engineers closely monitor the angle at which the tower leans. With careful measuring, they know that the point labeled *A* in the diagram at right is now 50 meters off the ground. Also, they determined that when a weight is dropped from point *A*, it lands 5 meters from the base of the tower, as shown in the diagram. Explore estimating the angle the tower is leaning using 4-1 Student eTool (Desmos).

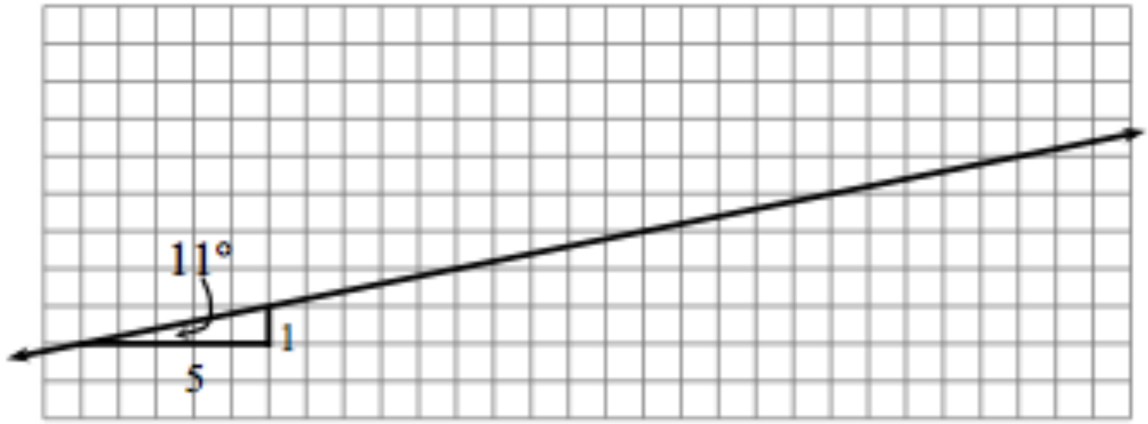
a) With the measurements provided above, what can you determine?

b) Can you determine the angle at which the tower leans? Why or why not?

c) At the end of Section 4.1, you will know how to find the angle for this situation and many others. However, at this point, how else can you describe the “lean” of the leaning tower?

4-2. PATTERNS IN SLOPE TRIANGLES

In order to find an angle (such as the angle at which the Leaning Tower of Pisa leans), you need to investigate the relationship between the angles and the sides of a right triangle. You will start by studying slope triangles in the graph shown below. Notice that one slope triangle has been drawn for you. Note: For the next several lessons angle measures will be rounded to the nearest degree.



a) Draw three new slope triangles on the line. Each should be a different size. Label each triangle with as much information as you can, such as its horizontal and vertical lengths and its angle measures.

b) Explain why all of the slope triangles on this line must be similar.

c) Since the triangles are similar, what does that tell you about the slope ratios?

d) Confirm your conclusion by writing the slope ratio for each triangle as a fraction, such as $\frac{\Delta y}{\Delta x}$. (Note: Δy represents the vertical change or "rise," while Δx represents the horizontal change or "run.") Then change the slope ratio into decimal form and compare.

4-3. Tara thinks she sees a pattern in these slope triangles, so she decides to make some changes in order to investigate whether or not the patterns remain true.

a) She asks, "*What if I draw a slope triangle on this line with* $\Delta y = 6$*? What would be the* Δx *of my triangle?*" Answer her question and explain how you figured it out.

b) "What if Δx is 40?" she wonders. "Then what is Δy?" Find Δy, and explain your reasoning.

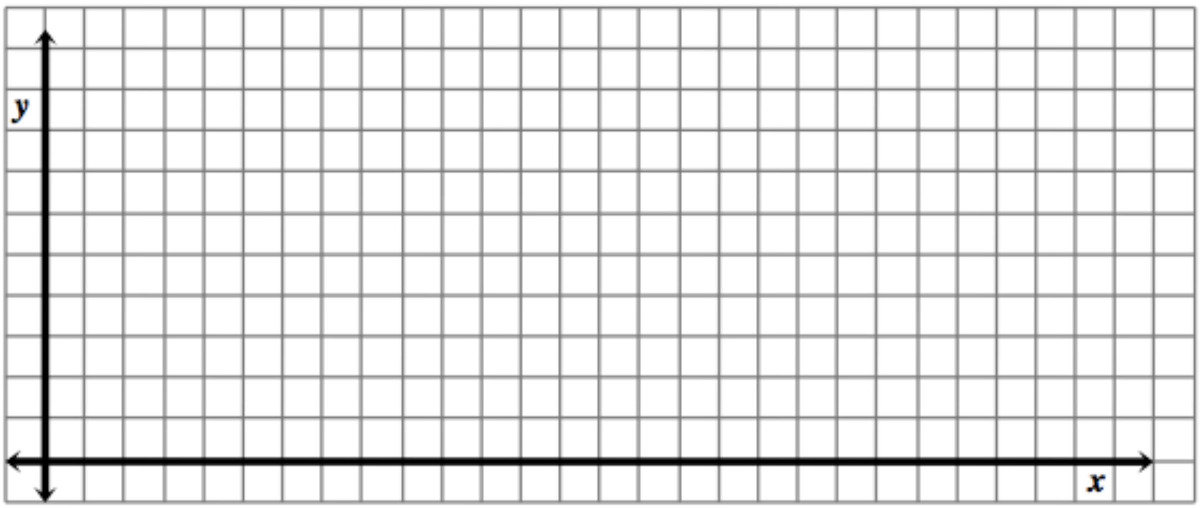
c) Tara wonders, "What if I draw a slope triangle on a different line? Can I still use the same ratio to find a missing Δx or Δy value?" Discuss this question with your team and explain to Tara what she could expect.

4-4. CHANGING LINES

In part (c) of problem 4-3, Tara asked, "*What if I draw my triangle on a different line?*" With your team, investigate what happens to the slope ratio and slope angle when the line is different. Use the graphs and your answers to the questions below to respond to Tara's question.

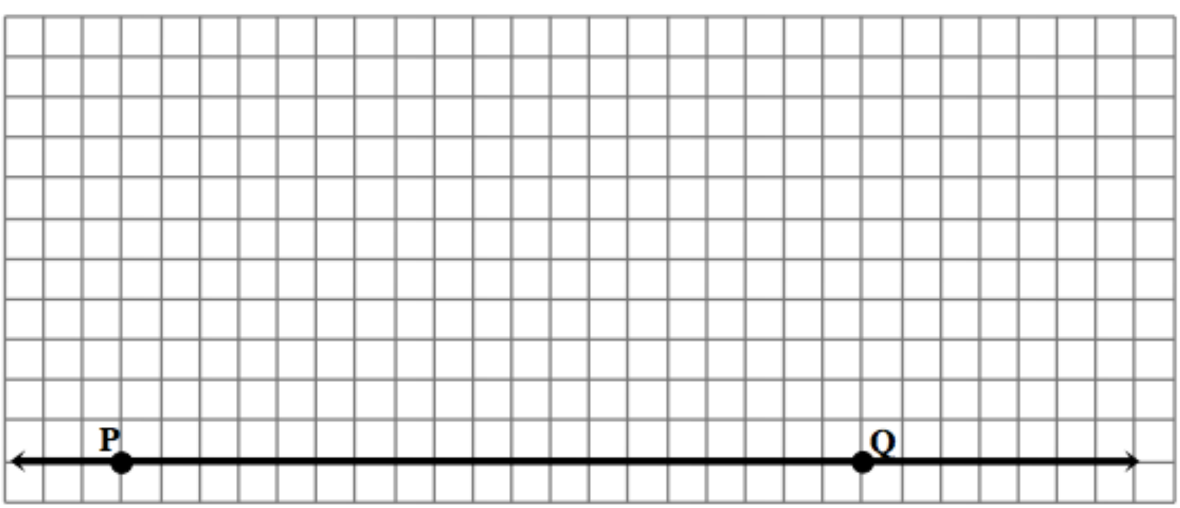
a) On graph A below, graph the line $y = \frac{2}{5}x$. What is the slope ratio for this line? What does the slope angle appear to be? Does the information about this line support or change your conclusion from part (c) of problem 4-3? Explain.

Graph A: (part (a) of problem 4-4)



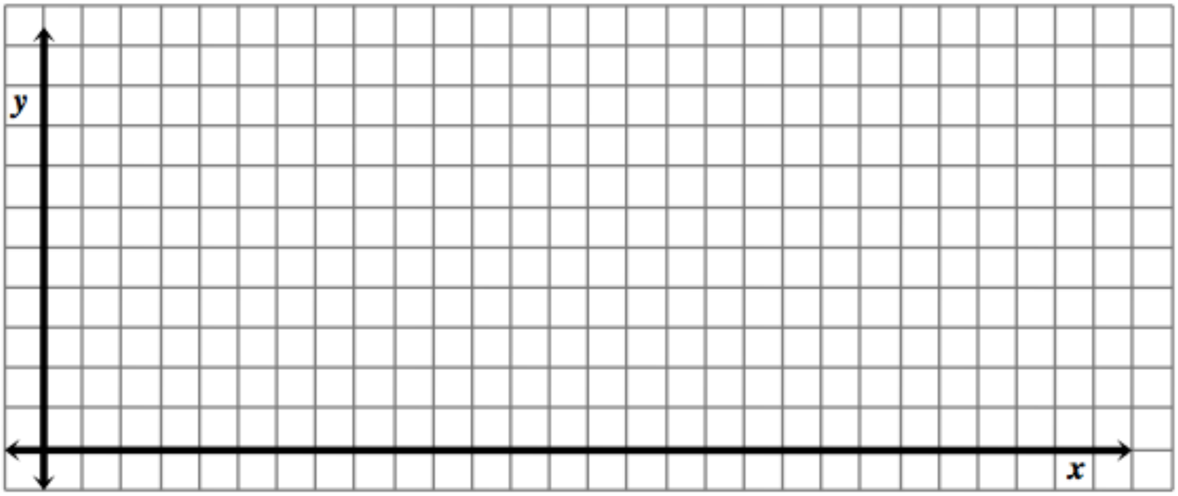
b) On graph B, you are going to create $\angle QPR$ so that it measures 18°. First, place your protractor so that point P is the vertex. Then find 18° and mark and label a new point, R. Draw ray $\overrightarrow{PR}$ to form $\angle QPR$. Find an approximate slope ratio for this line.

Graph B: (part (b) of problem 4-4)



c) Graph the line $y = x + 4$ on graph C. Draw a slope triangle and label its horizontal and vertical lengths. What is $\frac{\Delta y}{\Delta x}$ (the slope ratio)? What is the slope angle?

Graph C: (part (c) of problem 4-4)



4-5. TESTING CONJECTURES

The students in Ms. Coyner's class are writing conjectures based on their work today. As a team, decide if you agree or disagree with each of the conjectures below. Explain your reasoning.

a) All slope triangles have a ratio $\frac{1}{5}$.

b) If the slope ratio is $\frac{1}{5}$ then the slope angle is approximately 11°.

c) If the line has an 11° slope angle, then the slope ratio is approximately $\frac{1}{5}$.

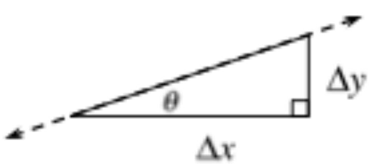
d) Different lines will have different slope angles and different slope ratios.

METHODS AND MEANINGS

MATH NOTES

Slope and Angle Notation

The slope of a line is the ratio of the vertical distance to the horizontal distance in a slope triangle formed by two points on a line. The vertical part of the triangle is called Δy, (read "change in *y*"), while the horizontal part of the triangle is called Δx (read "change in *x*"). Slope can then be written as $\frac{\Delta y}{\Delta x}$. Slope indicates both how steep the line is and its direction, upward or downward, left to right.



When a side length in a triangle is missing, that length is often assigned a variable from the English alphabet such as x, y, or z. However, sometimes you need to distinguish between an unknown side length and an unknown angle measure. With that in mind, mathematicians sometimes use Greek letters as variables for angle measurement. The most common variable for an angle is the Greek letter θ(*theta*), pronounced "THAY-tah." Two other Greek letters commonly used include α(*alpha*), and β(*beta*), pronounced "BAY-tah."

When a right triangle is oriented like a slope triangle, such as the one in the diagram above, the angle the line makes with the horizontal side of the triangle is called a **slope angle**.