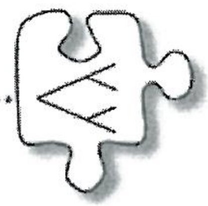
Name: Notes Period: ___________ Date: ____________

4.2.3 What model should I use?

Probability Models



In this lesson you will review ideas of probability as you use systematic lists, tree diagrams, and area models to account for all of the elements in a sample space, account for equally likely outcomes, and identify events. You will find that certain tools may work better for particular situations. In one problem a tree diagram or list might be most efficient, while in another problem an area model may be the best choice. As you work with your team, keep the following questions in mind:

What are the possible outcomes?
Are the outcomes equally likely?
Will a tree diagram, list, or area model help?
What is the probability for this event?

4-75. ROCK, PAPER, SCISSORS

Your team will play a variation of "Rock, Paper, Scissors" (sometimes called "Rochambeau") and record points. You will need to work in a team of four. Have one person act as recorder while the other three play the game.

a. List the names of the people in your team alphabetically. The first person on the list is Player A, the next is Player B, the third is Player C, and the fourth is the recorder. Write down who has each role.

b. Without playing the game, discuss with your team which player you think will receive the most points by the end of the game. Assign points as follows:
   a. Player A gets a point each time all three players match.
   b. Player B gets a point each time two of the three players match.
   c. Player C gets a point each time none of the players match.

c. Now play "Rock, Paper, Scissors" with your team at least 20 times. The recorder should record the winner for each round. Does this game seem fair?

d. Calculate the theoretical probability for each outcome (Player A, Player B, or Player C winning). Discuss this with your team and be prepared to share your results with the class.

e. Devise a plan to make this game fair.

4-76. There is a new game at the school fair called "Pick a Tile," in which the player reaches into two bags and chooses one square tile and one circular tile. The bag with squares contains three yellow, one blue, and two red squares. The bag with circles has one yellow and two red circles. In order to win the game (and a large stuffed animal), a player must choose one blue square and one red circle. Explore this game using the <u>*4-76 Student eTool*</u> (CPM).

Since it costs $2 to play the game, Marty and Gerri decided to calculate the probability of winning before deciding whether to play.

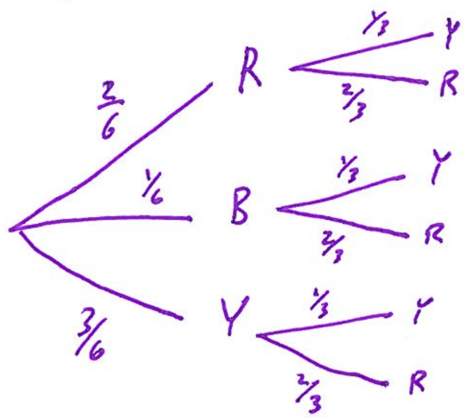
Gerri suggested making a systematic list of all the possible color combinations in the sample space, listing squares first then circles:

RY *BY* *YY*
RR *BR* *YR*

*"So,"*says Gerri, *"the answer is* $\frac{1}{6}$*."*

"That doesn't seem quite right," says Marty. *"There are more yellow squares than blue ones. I don't think the chance of getting a yellow square and a red circle should be the same as getting a blue square and a red circle."*

a. Make a tree diagram for this situation. Remember to take into account the duplicate tiles in the bags.



b. Find the probability of a player choosing the winning blue square-red circle combination.

$P(BR) = \frac{2}{18}$

c. Should Gerri and Marty play this game? Would you? Why or why not?

4-77. Now draw a probability area model for the "Pick a Tile" game in problem 4-76.

	$R: \frac{2}{3}$	$Y: \frac{1}{3}$
$Y: \frac{1}{2}$	$\frac{2}{6}$	$\frac{1}{6}$
$R: \frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$
$B: \frac{1}{6}$	$\frac{2}{18}$	$\frac{1}{18}$

a. Use the probability area model to calculate the probability of each possible color combination of a square and a circular tile.

b. Explain to Marty and Gerri why the probability area model is called an *area* model.

c. Discuss which model you preferred using to solve the "Pick a Tile" problem with your team. What are your reasons for your preference?

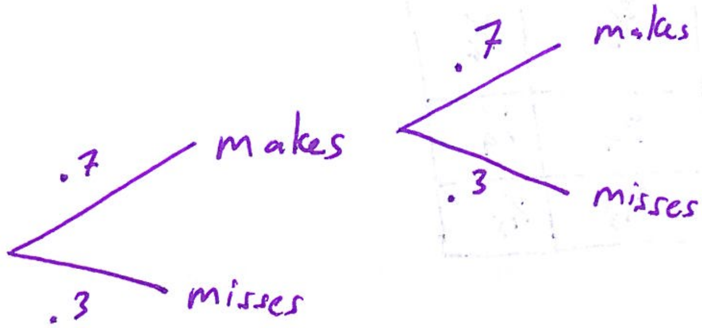
d. Could you have used the area model for the "Rock, Paper, Scissors" problem? Explain why or why not.

No, area only works with 2 choices

4-78. BASKETBALL: Shooting One-and-One Free Throws
Rimshot McGee has a 70% free throw average. The opposing team is ahead by one point. Rimshot is at the foul line in a one-and-one situation with just seconds left in the game. (A one-and-one situation means that the player shoots a free throw. If they make the shot, they are allowed to shoot another. If they miss the first shot, they get no second shot. Each shot made is worth one point.)

a. First, take a guess. What do you think is the most likely outcome for Rimshot: zero points, one point, or two points?

b. Draw a tree diagram to represent this situation.



2 points

$(.7)(.7) = 0.49$

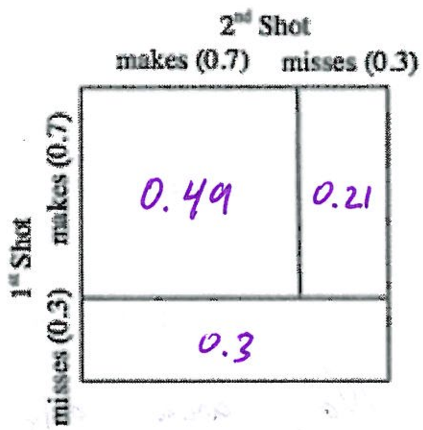
1 point

$(.7)(.3) = 0.21$

0 points

0.3

c. Jeremy is working on the problem with Jenna and he remembers that area models are sometimes useful for solving problems related to probability. They set up the probability area model below. Discuss this model with your team. Which part of the model represents Rimshot getting one point? How can you use the model to help calculate the probability that Rimshot will get exactly one point?



d. Use either your tree diagram or the area model to help you calculate the probabilities that Rimshot will get either 0 or 2 points. What is the most likely of the three outcomes?

2 points

4-79. With your team, examine the probability area model from problem 4-78.

What are the dimensions of the large rectangle? Explain why these dimensions make sense.
What is the total area of the model? Express the area as a product of the dimensions and as a sum of the parts.
What events are represented by the entire area model?

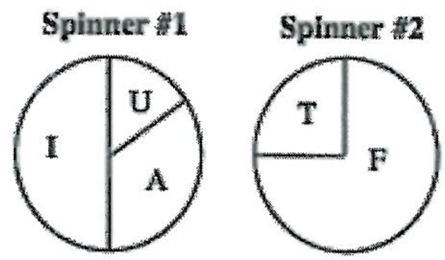
4-80. This Learning Log extends the entry that you made in problem 4-68. In that entry you described the various ways of representing complete sample spaces and showed how to use each method to find probabilities.
Expand upon your entry. Are there any conditions under which certain methods to represent the sample space can or cannot be used? Which methods seem most versatile? Why? Title this entry "Conditions For Using Probability Methods" and include today's date.

Probability Models

When all the possible outcomes of a probabilistic event are *equally likely*, you can calculate probabilities as follows:

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

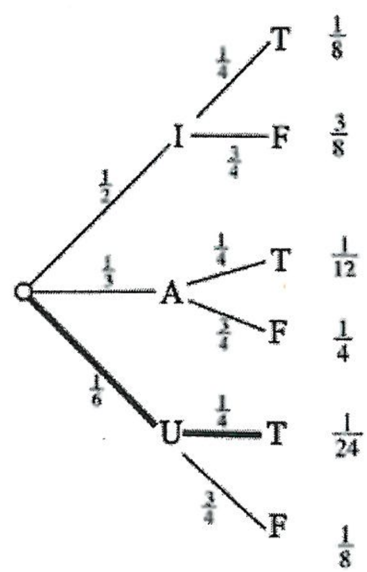
But suppose you spin the two spinners shown at right. These outcomes are not all equally likely so another model is needed to calculate probabilities of outcomes.



Spinner #2 \ Spinner #1	I $\left(\frac{1}{2}\right)$	A $\left(\frac{1}{3}\right)$	U $\left(\frac{1}{6}\right)$
T $\left(\frac{1}{4}\right)$	IT $\left(\frac{1}{8}\right)$	AT $\left(\frac{1}{12}\right)$	UT $\left(\frac{1}{24}\right)$
F $\left(\frac{3}{4}\right)$	IF $\left(\frac{3}{8}\right)$	AF $\left(\frac{1}{4}\right)$	UF $\left(\frac{1}{8}\right)$

A **probability area model** is practical if there are exactly two probabilistic situations and they are independent. The outcomes of one probabilistic situation are across the top of the table, and the outcomes of the other are on the left. The smaller rectangles are the sample space. Then the probability for an outcome is the area of the rectangle. For example, the probability of spinning "UT" is $\frac{1}{6}\cdot\frac{1}{4}=\frac{1}{24}$. Notice that the area (the probability) of the large overall square is 1.

A **tree diagram** can be used even if there are more than two probabilistic situations, and the events can be independent or not. In this model, the ends of the branches indicate outcomes of probabilistic situations, and the branches show the probability of each event. For example, in the tree diagram at right the first branching point represents Spinner #1 with outcomes "I" "A" or "U". The numbers on the branch represent the probability that a letter occurs.



The numbers at the far right of the table represent the probabilities of various outcomes. For example, the probability of spinning "U" and "T" can be found at the end of the bold branch of the tree. This probability, $\frac{1}{24}$, can be found by multiplying the fractions that appear on the bold branches.