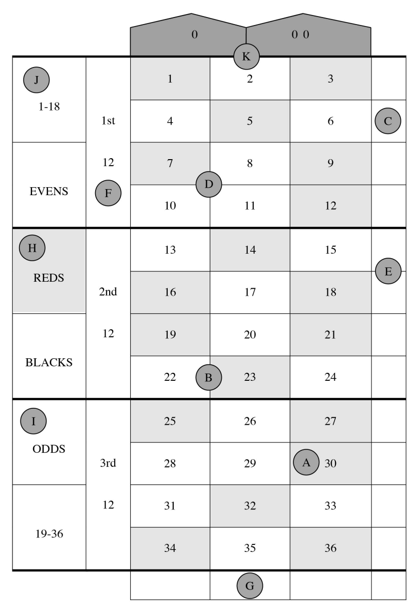
4.2.4 What if both events happen?

Unions, Intersections, and Complements

In the mid 1600's, a French nobleman, the Chevalier de Mere, was wondering why he was losing money on a bet that he thought was a sure winner. He asked the mathematician Blaise Pascal, who consulted with another mathematician, Pierre de Fermat. Together they solved the problem, and this work provided a beginning for the development of probability theory. Since argument over the analysis of a dice game provided a basis for the study of this important area of mathematics, casino games are a reasonable place to continue to investigate and clarify the ideas and language of probability.

As one of the simplest casino games to analyze, roulette is a good place to start. In American roulette the bettor places a bet, the croupier (game manager) spins the wheel and drops the ball and then everyone waits for the ball to land in one of 38 slots. The 38 slots on the wheel are numbered 00, 0, 1, 2, 3, …, 36. Eighteen of the numbers are red and eighteen are black; 0 and 00 are green. (In French roulette, also known as Monte Carlo, there is no 00, so there are only 37 slots on the wheel.)
Before the ball is dropped, players place their chips on the roulette layout, shown below. Bets can be placed on:

- A single number;
- Two numbers by placing the chip on the line between them;
- Three numbers by placing a chip on the line at the edge of a row of three;
- Four numbers by placing the chip where the four corners meet;
- Five numbers (0, 00, 1, 2, 3);
- Six numbers by placing the bet at an intersection at the edge;
- A column, the 1st twelve, 2nd twelve, or 3rd twelve;
- Even numbers;
- Odd numbers;
- 1-18;
- 19-36;
- Red numbers; or,
- Black numbers



Note: The lightly shaded numbers, 1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, and 36 are red.

4-87. Obtain a Lesson 4.2.4A Resource Page from your teacher. On the resource page, the "chips" A through K represent possible bets that could be made.

a. What is the sample space for one spin of the roulette wheel?

b. Are the outcomes equally likely?

c. A subset (smaller set) of outcomes from the sample space is called an event. For example, chip A represents the event {30}, and chip B represents the event {22, 23}. On your resource page make a list of events and their probabilities for Chips A-K.

4-88. Some roulette players like to make two (or more) bets at the same time. A bettor places a chip on the event {7, 8, 10, 11} and then another chip on the event {10, 11, 12, 13, 14, 15}. What numbers will allow the bettor to win both bets? Next find the bettor's chances of winning the bet of the first chip *and* winning the bet of the second chip on a single drop of the roulette ball. This is called the probability of the ball landing on a number that is in the **intersection of two events**.

4-89. When placing two different bets, most players are just hoping that they will win on one *or* the other of the two events. The player is betting on the **union** of two events.

a. Calculate the probability of winning either the bet on the event {7, 8, 10, 11} *or* the bet on the event {10, 11, 12, 13, 14, 15}. Think about the set of outcomes that will allow the bettor to win either of the bets. This set of outcomes is the union of the two events.

b. Calculate the probability of the union of {numbers in first column} and {"2^{nd} 12" numbers 13 through 24}.

c. One bettor's chip is on the event {13, 14, 15, 16, 17, 18} and another on {Reds}. What is the probability of the union of these events?

d. Explain your method for finding the probabilities in parts (a) through (c).

4-90. Viola described the following method for finding the probability for part (a) of problem 4-89:
"When I looked at the probability of either of two events, I knew that would include all of the numbers in both events, but sometimes some numbers might be counted twice. So, instead of just counting up all of the outcomes, I added the two probabilities together and then subtracted the probability of the overlapping events or numbers. So it's just ."

$$\frac{4}{38}+\frac{6}{38}-\frac{2}{38}=\frac{8}{38}$$

Does Viola's method always work? Why or why not? Is this the method that you used to do problem 4-89? If not show how to use Viola's method on one of the other parts of problem 4-89.

4-91. Viola's method of "adding the two probabilities and subtracting the probability of the overlapping event" is called the **Addition Rule** and can be written:

$$P(A \textit{ or } B) = P(A) + P(B) - P(A \textit{ and } B)$$

You have already seen that any event that includes event A *or* event B can be called a union, and is said "A union B." The event where both events A *and* B occur together is called an intersection. So the Addition Rule can also be written:

$$P(A \text{ union } B) = P(A) + P(B) - P(A \text{ intersection } B)$$

Use these ideas to do the following: A player places a chip on the event {1-18} and another on the event {Reds}. Consider the event {1-18} as event "A", and the event {Reds} as event "B." Clearly show two different ways to figure out the probability of the player winning one of the two bets.

4-92. On the Lesson 4.2.4A Resource Page, consider a player who puts a chip on both events "G" and "I."

a. How does the event {G *or* I} differ from the event {G *and* I}?

b. List the set of outcomes for the intersection of events G and I, and the set of outcomes for the union of events G or I.

c. Is the player who puts a chip on both G and I betting on the "or" or the "and?" Use both the counting method and the Addition Rule to find the probability that this player will win.

4-93. Wyatt places a bet on event G.

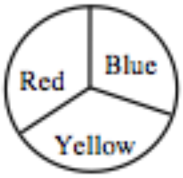
a. What is the probability that he will lose?

b. How did you calculate the probability of {not event G}?

c. Show another method for calculating the probability of the bettor losing on event G.

4-94. Sometimes it is easier to figure out the probability that something will *not* happen than the probability that it *will*. When finding the probability that something will not happen, you are looking at the **complement** of an event. The complement is the set of all outcomes in the sample space that are not included in the event.

Show two ways to solve the problem below, then decide which way you prefer and explain why.



a. Crystal is spinning the spinner at right and claims she has a good chance of having the spinner land on red at least once in three tries. What is the probability that the spinner will land on red at least once in three tries?

b. If the probability of an event A is represented symbolically as P(A), how can you symbolically represent the probability of the complement of event A?

METHODS AND MEANINGS

MATH NOTES

Probability Models

When all the possible outcomes of a probabilistic event have the same probability, probabilities can be calculated by listing the possible outcomes in a **systematic list**. However, when some outcomes are more probable than others, a more sophisticated model is required to calculate probabilities.

A smaller set of outcomes from a sample space is called an **event**.

The **complement** of an event is all the outcomes in the sample space that are not in the original event. For example, the complement of {drawing a spade} would be all the hearts, diamonds, and clubs, represented as the complement of {drawing a spade}.

The **intersection of two events** is the event in which *both* the first event *and* the second event occur. The intersection of the events {drawing a spade} and {face card} would be {J♠, Q♠, K♠} because these three cards are in both the event {drawing a spade} *and* the event {face card}.

The **union** of two events is the event in which the first event *or* the second event (or both) occur. The union of the events {drawing a spade} or a {face card} has 22 outcomes.

The probability of *equally likely* events can be found by: $\dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

The probability of {drawing a spade} or {drawing a face card} is $\frac{22}{52}$ *because there are 22 cards in the union and 52 cards in the sample space.*

Alternatively, the probability of the union of two events can be found by using the **Addition Rule**:

P(A or B) = P(A) + P(B) – P(A and B)

or

P(A union B) = P(A) + P(B) – P(A intersection B)

If you let event A be {drawing a spade} and event B be {drawing a face card},

P(A) = P(spade) = , P(B) = P(face card) = $\frac{12}{52}$, P(A and B) = P(spade and face card) = $\frac{3}{52}$.

Then, the probability of drawing a spade *or* a face card is: P(A or B) = P(A) + P(B) – P(A and B) = $\frac{13}{52}+\frac{12}{52}-\frac{3}{52}=\frac{22}{52}$.