Name: Notes Period: ________ Date: __________

4.2.1 How can I represent it?

Using an Area Model

"and" = multiply
"or" = add

$\frac{\#\text{ of success}}{\#\text{ of outcomes}} = \text{probability}$

In previous courses you studied probability, which is a measure of the chance that a particular event will occur. In the next few lessons you will encounter a variety of situations that require probability calculations. You will develop new probability tools to help you analyze these situations. The next two lessons focus on tools for listing *all* the possible outcomes of a probability situation, called a **sample space.**

In homework, you have practiced determining probabilities in situations where each outcome you listed had an equal probability of occurring. But what if a game is biased so that some outcomes are more likely than others? How can you represent biased games? Today you will learn a new tool to analyze more complicated situations of chance, called an area model.

4-53. IT'S IN THE GENES

Can you bend your thumb backwards at the middle joint to make an angle, like the example at right? Or does your thumb remain straight? The ability to bend your thumb back is thought to rely on a single gene.

Example of a thumb that can bend backwards at the joint.

What about your tongue? If you can roll your tongue into a "U" shape, you probably have a special gene that enables you to do this.

Assume that half of the U.S. population can bend their thumbs backwards and that half can roll their tongues. Also assume that these genes are independent (in other words, having one gene does not affect whether or not you have the other) and randomly distributed (spread out) throughout the population. Then the sample space of these genetic traits can be organized in a table like the one below.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$P(\text{both}) = \frac{1}{4}$

$P($

Roll Tongue?	Bend thumb? Yes $\frac{1}{2}$	Bend thumb? No $\frac{1}{2}$
Yes $\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
No $\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

a. According to this table, what is the probability that a random person from the U.S. has both special traits? That is, what is the chance that he or she can roll his or her tongue *and* bend his or her thumb back?

$P(\text{both}) = \frac{1}{4}$

b. According to this table, what is the probability that a random person has only one of these special traits? Justify your conclusion.

P(bend thumb or roll tongue)

*Remember, "or" is add

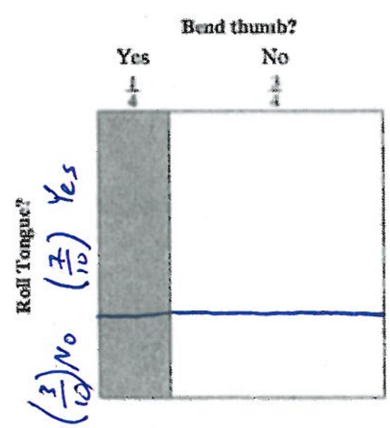
$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

c. This table is useful because every cell in the table is equally likely. Therefore, each possible outcome, such as being able to bend your thumb but not roll your tongue, has a $\frac{1}{4}$ probability. However, this table assumes that half the population can bend their thumbs backwards, but in reality only about $\frac{1}{4}$ of the U.S. population can bend their thumbs backwards and $\frac{3}{4}$ cannot. It also turns out that a lot more (about $\frac{7}{10}$) of the population can roll their tongues. How can this table be adjusted to represent these percentages? Discuss this with your team and be prepared to share your ideas with the class.

4-54. USING AN AREA MODEL

One way to represent a sample space that has outcomes that are not equally likely is by using a **probability area model**. An area model uses a large square with an area of 1. The square is subdivided into smaller pieces to represent all possible outcomes in the sample space. The area of each outcome is the probability that the outcome will occur.

For example, if $\frac{1}{4}$ of the U.S. population can bend their thumbs back, then the column representing this ability should take only one-fourth of the square's width, as shown below.



a. How should the diagram be altered to show that $\frac{7}{10}$ of the U.S. can roll their tongues? Copy this diagram on your paper and add two rows to represent this probability.

b. The relative probabilities for different outcomes are represented by the areas of the regions. For example, the portion of the probability area model representing people with both special traits is a rectangle with a width of $\frac{1}{4}$ and a height of $\frac{7}{10}$

c. What is the area of this rectangle? This area tells you the probability that a random person in the U.S. has both traits.

$\frac{7}{10} + \frac{3}{10} = 1$ $\qquad 1 \times 1 = \boxed{1 \leftarrow \text{total area}}$

$\frac{1}{4} + \frac{3}{4} = 1$

* Remember, "and" means multiply

P(bend ~~tongue~~ thumb and roll tongue)

$\frac{1}{4} \times \frac{7}{10} = \boxed{\frac{7}{40}} = \boxed{0.175}$

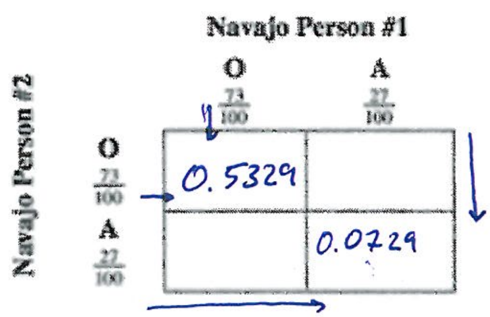
d. What is the probability that a randomly selected person can roll his or her tongue but not bend his or her thumb back? Show how you got this probability.

P(can roll tongue and cannot bend thumb)

$\frac{7}{10} \times \frac{3}{4} = \boxed{\frac{21}{40} = 0.525}$

4-55. PROBABILITIES IN VEIN

You and your best friend may not only look different, you may also have different types of blood! For instance, members of the American Navajo population can be classified into two groups: 73% percent (73 out of 100) of the Navajo population has type "O" blood, while 27% (27 out of 100) has type "A" blood. (Blood types describe certain chemicals, called "antigens," that are found in a person's blood.)



a. Suppose you select two Navajo individuals at random. What is the probability that both individuals have type "A" blood? This time, drawing an area model that is exactly to scale would be challenging. A probability area model (like the one above) is still useful because it will still allow you to calculate the individual areas, even without drawing it to scale. Copy and complete this "generic" probability area model.

b. What is the probability that two Navajo individuals selected at random have the same blood type?

P(O and O) $\qquad$ P(A and A)

$\frac{73}{100} \times \frac{73}{100} = 0.5329$ $\qquad \frac{27}{100} \times \frac{27}{100} = 0.0729$

P(OO or AA)

$0.5329 + 0.0729 = \boxed{0.6058}$

4-56. SHIPWRECKED!

Zack and Nick (both from the U.S.) are shipwrecked on a desert island! Zack has been injured and is losing blood rapidly, and Nick is the only person around to give him a transfusion.

Unlike the Navajo you learned of in problem 4-55, most populations are classified into four blood types: O, A, B, and AB. For example, in the U.S., 45% of people have type O blood, 40% have type A, 11% have type B, and 4% have type AB (according to the American Red Cross, 2004). While there are other ways in which people's blood can differ, this problem will only take into account these four blood types.

a. Make a probability area model representing the blood types in this problem. List Nick's possible blood types along the top of the model and Zack's possible blood types along the side.

Zack \ Nick	A (.40)	B (.11)	O (.45)	AB (0.04)
A (.40)	.16		.18	
B (.11)		.0121	.0495	
O (.45)			.2025	
AB (.04)	.016	.0044	.018	.0016

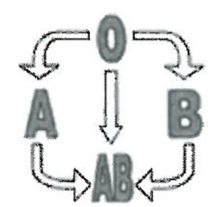
b. What is the probability that Zack and Nick have the same blood type?

$P(A \text{ and } A) = .4 \times .4 = .16$ $\quad P(O \text{ and } O) = .45 \times .45 = .2025$

$P(B \text{ and } B) = .11 \times .11 = .0121$ $\quad P(AB \text{ and } AB) = .04 \times .04 = .0016$

$.16 + .0121 + .2025 + .0016 = \boxed{.3762}$

c. Luckily, two people do not have to have the same blood type for the receiver of blood to survive a transfusion. Other combinations will also work, as shown in the diagram below. Assuming that their blood is compatible in other ways, a donor with type O blood can donate to receivers with type O, A, B, or AB, while a donor with type A blood can donate to a receiver with A or AB. A donor with type B blood can donate to a receiver with B or AB, and a donor with type AB blood can donate only to AB receivers.



Assuming that Nick's blood is compatible with Zack's in other ways, determine the probability that he has a type of blood that can save Zack's life!

$0.3762 + 0.18 + 0.0495 + 0.018 + 0.0044 + 0.016$

$= \boxed{.6441}$

4-57. You made a critical assumption in problem 4-56 when you made a probability area model and multiplied the probabilities.

a. Blood type is affected by genetic inheritance. What if Zack and Nick were related to each other? What if they were brothers or father and son? How could that affect the possible outcomes?

b. What has to be true in order to assume a probability area model will give an accurate theoretical probability?

Solving a Quadratic Equation:

In a previous course, you learned how to solve **quadratic equations** (equations that can be written in the form $ax^2 + bx + c = 0$). Review two methods for solving quadratic equations below. Some quadratic equations can be solved by **factoring** and using the **Zero Product Property.** For example, because $x^2 - 3x - 10 = (x - 5)(x + 2)$, the quadratic equation $x^2 - 3x - 10 = 0$ can be rewritten as $(x - 5)(x + 2) = 0$. The Zero Product Property states that *if* $ab = 0$, then $a = 0$ or $b = 0$. So, if $(x - 5)(x + 2) = 0$, then $x - 5 = 0$ or $x + 2 = 0$. Therefore, $x = 5$ or $x = -2$.
Another method for solving quadratic equations is the **Quadratic Formula.** This method is particularly helpful for solving quadratic equations that are difficult or impossible to factor. Before using the Quadratic Formula, the quadratic equation you want to solve must be in standard form, that is, written as$ax^2 + bx + c = 0$.
In this form, a is the coefficient of the x^2 term, b is the coefficient of the x term, and c is the constant term. The Quadratic Formula states:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula gives two possible answers due to the "±" symbol. This symbol (read as "plus or minus") is shorthand notation that tells us to calculate the formula twice: once using addition and once using subtraction. Therefore, every Quadratic Formula problem must be simplified twice to give:

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

To solve $x^2 - 3x - 10 = 0$ using the Quadratic Formula, substitute $a = 1$, $b = -3$, and $c = -10$ into the formula, as shown below.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)} \rightarrow \frac{3 \pm \sqrt{49}}{2} \rightarrow \frac{3 \pm 7}{2} \rightarrow x = 5 \text{ or } x = -2$$